SUBJECT : MATHEMATICS (SET-I)

Time : 3 Hrs. **M.M.: 100**

General Instructions :

(i) Question nos. 1 to 6 carry 1 mark each.

(ii) Question nos. 7 to 19 carry 4 marks each.

(iii) Question nos. 20 to 26 carry 6 marks each.

(iv) Use of calculator is not permitted.

SECTION-A

Q1. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then for what value of α is A an identity matrix?

Q2. What is the value of $\begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix}$?

Q3. Evaluate : $\int \frac{\log x}{x}\, dx$

Q4. What is the degree of the following differential equation?

$$5x\left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} - 6y = \log x$$

Q5. Write a vector of magnitude 15 units in the direction of vector $\hat{i} - 2\hat{j} + 2\hat{k}$.

Q6. Evaluate $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

SECTION-B

Q7. On a multiple choice examination with three possible answers (out of which only one is correct) for each of five questions. What is the probability that a candidate would get four or more correct answers just by guessing?

Q8. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $(2\vec{a} + \vec{b})$ and $(\vec{a} - 3\vec{b})$ respectively, externally in the ratio 1:2. Also, show that P is the mid point of the line segment RQ.

Q9. Evaluate : $\int \frac{(2x^2+1)}{x^2(x^2+4)} dx$

Q10. Using elementary row transormations, find the inverse of the following matrix : $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

Q11. Let Z be the set of all integers and R be the relation on Z defined as R = {(a, b) : a, b ε Z; (a - b) is divisible by 5}. Prove that R is an equivalence relation.

Q12. Prove the following :

$$\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x \in (0, 1)$$

Q13. Show that the function f defined as follows, is continuous at $x = 2$, but not differentiable there at :

$$f(x) = \begin{cases} 3x - 2 & ; \quad 0 < x \le 1 \\ 2x^2 - x & ; \quad 1 < x \le 2 \\ 5x - 4 & ; \quad x > 2 \end{cases}$$

Q14. Find $\frac{dy}{dx}$; if $y = \sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right]$

Q15. Evaluate : $\int e^x\left(\frac{\sin 4x - 4}{1 - \cos 4x}\right) dx$

Q16. Evaluate : $\int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$

Q17. Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to the y-coordinate of the point.

Q18. Find the general solution of the differential equation satisfying the given conditions :

$$x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x$$

Q19. Find the particular solution of the differential equation.

$x^2\, dy + (xy + y^2)\, dx = 0$; $y = 1$ when $x = 1$

SECTION-C

Q20. A small firm manufacturers gold rings and chains. The total number of rings and chains manufactured per day is atmost 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is ₹ 300 and that on a chain is ₹ 190. Find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit. Make it as L.P.P. and solve it graphically.

Q21. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn at random and are found to be both clubs. Find the probability of the lost card being of clubs.

Q22. Evaluate as limit of sum : $\int_{-1}^{2} \left(x^2 - 3x\right) dx$

Q23. Using integration, find the area of the region bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$

Q24. Show that the right circular cylinder, open at the top, and of given surface area and maximum volume is such that its height is equal to the radius of the base.

Q25. Find the values of x for which $f(x) = [x(x-2)]^2$ is an increasing function. Also, find the points on the curve, where the tangent is parallel to x-axis.

Q26. Using properties of determinants, show the following :

$$\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc\,(a+b+c)^3$$